

# Symmetry-breaking corrections to heavy meson form-factor relations

Benjamín Grinstein and Dan Pirjol<sup>1</sup>

<sup>1</sup>*Department of Physics, UCSD, 9500 Gilman Drive, La Jolla, CA 92093*

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## Abstract

In the heavy quark limit, the form factors for semileptonic and rare radiative  $B$  decays into light mesons are related by heavy quark spin symmetry. Here we compute the leading corrections of order  $\Lambda/m_Q$  to these symmetry relations, showing also how to include hard gluon effects systematically to any order in  $\alpha_s(m_Q)$ . The subleading correction to the form factor relation for  $B \rightarrow \pi$  can be computed exactly in the soft pion limit. For decays into vector mesons, the matrix elements of two local dimension-4 operators are needed, one of which vanishes in the constituent quark model. A few applications are briefly discussed.

A precise determination of the Cabibbo-Kobayashi-Maskawa matrix elements using exclusive heavy meson decays depends in a crucial way on our theoretical control over strong interaction effects in these processes. The heavy quark effective theory (HQET) [1] provides a systematic framework within which the relevant matrix elements can be studied in an expansion in powers of  $1/m_Q$ , the inverse heavy quark mass. We focus in this Letter on form factors for heavy-light transitions, which are relevant for semileptonic and rare decays such as  $\bar{B} \rightarrow \rho \ell \bar{\nu}_\ell$  and  $\bar{B} \rightarrow K^* \ell^+ \ell^-$ .

Heavy quark spin symmetry has been used to relate heavy-to-light form factors corresponding to different currents [2, 3]. These relations hold true in the kinematical region where the energy of the final light hadron  $E$  in the rest frame of the  $B$  meson is not too large. At large recoil  $E \gg \Lambda_{QCD}$ , a new symmetry comes into play, further simplifying the structure of the form factors [4]. At the same time,  $\mathcal{O}(\alpha_s(m_Q))$  perturbative corrections due to the exchange of hard gluons become important. They can be computed in a systematic way in perturbation theory, and the corresponding  $\mathcal{O}(\alpha_s)$  corrections have been evaluated in [5, 6].

In both these kinematical regions there are corrections of order  $\Lambda/m_Q$  to the symmetry relations for the form factors, arising from subleading operators in the HQET. The complete expansion of the individual form factors at order  $\Lambda/m_Q$  involves in general a large number of both local and nonlocal contributions and appears to preserve little predictive power. Such analyses have been given in [7] for  $\bar{B} \rightarrow P$  and in [8] for the  $\bar{B} \rightarrow V$  form factors, where  $P$  and  $V$  are pseudoscalar and vector light mesons, respectively. We show in this Letter that, with certain modifications required to include hard gluon corrections, the symmetry relations can be simply extended to subleading order. The subleading corrections have a simple form and can be expressed only in terms of lowest order form factors and the matrix elements of two dimension-4 local operators.

The hadronic matrix elements relevant for  $\bar{B}$  decays into a light meson are parametrized in terms of form factors. There are three form factors for decays into a pseudoscalar, relevant for semileptonic decays (e.g.  $B \rightarrow \pi e \nu$ )

$$\langle P(p') | \bar{q} \gamma_\mu b | \bar{B}(p) \rangle = f_+(p_\mu + p'_\mu) + f_-(p_\mu - p'_\mu) \quad (1)$$

and rare decays (e.g.  $B \rightarrow K e^+ e^-$ )

$$\langle P(p') | \bar{q} \sigma_{\mu\nu} b | \bar{B}(p) \rangle = i s [(p_\mu + p'_\mu)(p_\nu - p'_\nu) - (p_\nu + p'_\nu)(p_\mu - p'_\mu)] \quad (2)$$

The  $\bar{B}$  decays into a light vector meson are parametrized by a total of seven form factors, four of which appear in semileptonic decay matrix elements

$$\langle V(p', \epsilon) | \bar{q} \gamma_\mu b | \bar{B}(p) \rangle = i g(q^2) \varepsilon_{\mu\nu\lambda\sigma} \epsilon_\nu^* (p + p')_\lambda (p - p')_\sigma \quad (3)$$

$$\begin{aligned} \langle V(p', \epsilon) | \bar{q} \gamma_\mu \gamma_5 b | \bar{B}(p) \rangle &= f(q^2) \epsilon_\mu^* \\ &+ a_+(q^2) (\epsilon^* \cdot p) (p + p')_\mu + a_-(q^2) (\epsilon^* \cdot p) (p - p')_\mu. \end{aligned} \quad (4)$$

and three form factors in rare radiative decays

$$\begin{aligned} \langle V(p', \epsilon) | \bar{q} \sigma_{\mu\nu} b | \bar{B}(p) \rangle &= g_+(q^2) \varepsilon_{\mu\nu\lambda\sigma} \epsilon_\lambda^* (p + p')_\sigma + g_-(q^2) \varepsilon_{\mu\nu\lambda\sigma} \epsilon_\lambda^* (p - p')_\sigma \\ &+ h(q^2) \varepsilon_{\mu\nu\lambda\sigma} (p + p')_\lambda (p - p')_\sigma (\epsilon^* \cdot p). \end{aligned} \quad (5)$$

While the vector and axial form factors are renormalization scale invariant, the form factors of the tensor current have a nontrivial scale dependence. Throughout in the following it will be understood implicitly that the scale used to define these form factors is  $\mu = m_b$ .

Counting powers of  $m_b$  coming from kinematical factors and the usual relativistic normalization of the  $|\bar{B}(p)\rangle$  states gives the heavy quark mass scaling properties of these form factors [2]

$$f_+(y) + f_-(y) \propto m_b^{-1/2}, \quad f_+(y) - f_-(y) \propto m_b^{1/2}, \quad s(y) \propto m_b^{-1/2} \quad (6)$$

and

$$f(y) \propto m_b^{1/2}, \quad g(y) \propto m_b^{-1/2}, \quad a_+(y) - a_-(y) \propto m_b^{-1/2}, \quad a_+(y) + a_-(y) \propto m_b^{-3/2} \quad (7)$$

$$g_+(y) - g_-(y) \propto m_b^{1/2}, \quad g_+(y) + g_-(y) \propto m_b^{-1/2}, \quad h \propto m_b^{-3/2}. \quad (8)$$

These scaling relations hold at a fixed value of the light meson energy in the rest frame of  $B$ ,  $E_V = m_V y$ . Therefore for a consistent  $m_b$  power counting, we will use  $y = p \cdot p' / (m_B m_V)$  as argument for the form factors, instead of  $q^2$ . These parameters are related as  $q^2 = m_B^2 + m_V^2 - 2m_B m_V y$ . The range of  $y$  probed in semileptonic  $D$  decays is  $1 \leq y \leq y_D^{\max} = \frac{m_D^2 + m_V^2}{2m_D m_V} \simeq 1.3$ , whereas in  $B$  decays the corresponding range is  $1 \leq y \leq 3.0$ .

For values of  $y = v \cdot v'$  not too far away from the zero recoil point  $y = 1$ , heavy quark spin symmetry can be used to relate some of the form factors in (1)-(5) [2, 3]. There is one relation among the  $B \rightarrow P$  form factors

$$(P-1) : \quad f_+(y) - f_-(y) - 2m_B s(y) = \mathcal{O}(m_b^{-1/2}) \quad (9)$$

and three relations among the  $B \rightarrow V$  form factors

$$(V-1) : \quad g_+(y) - g_-(y) = -2m_B g(y) + \mathcal{O}(m_b^{-1/2}) \quad (10)$$

$$(V-2) : \quad g_+(y) + g_-(y) = 2m_V y g(y) + \frac{1}{m_B} f(y) + \mathcal{O}(m_b^{-3/2}) \quad (11)$$

$$(V-3) : \quad a_+(y) - a_-(y) = 2g(y) - 2m_B h + \mathcal{O}(m_b^{-3/2}). \quad (12)$$

In the following we will review the derivation of these relations using a method which will allow us to include hard gluon corrections to all orders in  $\alpha_s(m_b)$ , and to compute the leading  $\mathcal{O}(\Lambda/m_b)$  corrections to them.

We illustrate the principle of the method on the example of Eq. (9), and start by considering the two currents  $J_\mu = \bar{q}\gamma_\mu b$  and  $J'_\mu = i\bar{q}\sigma_{\mu\nu}v^\nu b$ . Both of them are matched in HQET onto operators of the general form

$$J_\mu^{(\prime)} = \bar{q}\Gamma_\mu^{(\prime)}b = c_0^{(\prime)}(\mu)\bar{q}\gamma_\mu h_v + c_1^{(\prime)}(\mu)\bar{q}v_\mu h_v + \frac{1}{2m_b} \sum_{i=1}^6 b_i^{(\prime)}(\mu)\mathcal{J}_\mu^i, \quad (13)$$

with  $\mathcal{J}_\mu^i$  the most general set of dimension-4 operators with the same transformation properties as the corresponding QCD current. Restricting ourselves to operators which do not vanish by the equations of motion, a complete basis can be chosen as

$$\begin{aligned} \mathcal{J}_\mu^1 &= \bar{q}\gamma_\mu i \not{p} h_v, & \mathcal{J}_\mu^2 &= \bar{q}v_\mu i \not{p} h_v, & \mathcal{J}_\mu^3 &= \bar{q}i D_\mu h_v, \\ \mathcal{J}_\mu^4 &= v \cdot i \partial (\bar{q}\gamma_\mu h_v), & \mathcal{J}_\mu^5 &= v \cdot i \partial (\bar{q}v_\mu h_v), & \mathcal{J}_\mu^6 &= i \partial_\mu (\bar{q}h_v). \end{aligned} \quad (14)$$

The Wilson coefficients of the dimension-3 operators  $c_{0,1}^{(\prime)}(m_b)$  at the matching scale  $c_i(m_b)$  are known to NNLO order [13]. For  $J_\mu$  they are given by

$$c_0(m_b) = 1 - \frac{\alpha_s C_F}{\pi} - (11.40 - 0.79n_\ell + 0.09) \left( \frac{\alpha_s}{\pi} \right)^2 \quad (15)$$

$$c_1(m_b) = \frac{\alpha_s C_F}{2\pi} + (7.30 - 0.35n_\ell + 0.02) \left( \frac{\alpha_s}{\pi} \right)^2, \quad (16)$$

and for  $J'_\mu$  by

$$c'_0(m_b) = -c'_1(m_b) = -\left(1 - \frac{\alpha_s C_F}{\pi} - (16.09 - 1.13n_\ell + 0.13) \left( \frac{\alpha_s}{\pi} \right)^2\right). \quad (17)$$

The corresponding Wilson coefficients of the dimension-4 operators are given at tree level by

$$\begin{aligned} b_1(m_b) &= 1, & b_{2-6}(m_b) &= 0 \\ b'_1(m_b) &= b'_2(m_b) = 1, & b'_{3-6}(m_b) &= 0. \end{aligned} \quad (18)$$

In addition to the local corrections coming from the matrix elements of the  $\mathcal{J}_\mu^i$  operators, there are also nonlocal subleading  $1/m_b$  operators which appear as time-ordered products of the leading dimension-3 currents with the  $1/m_b$  terms in the HQET Lagrangian

$$\mathcal{L}_{\text{kin}} = \bar{h}_v (iD)^2 h_v, \quad \mathcal{L}_{\text{mag}} = \frac{1}{2} \bar{h}_v g \sigma_{\mu\nu} F^{\mu\nu} h_v. \quad (19)$$

Reparametrization invariance [9] gives the relation  $c_0(\mu) = b_1(\mu)$  among the Wilson coefficients of the current  $J_\mu$ , which should be satisfied to all orders in  $\alpha_s$ . No such constraints exist among the Wilson coefficients of  $J'_\mu$ , for which reparametrization invariance has been broken explicitly by defining it in terms of  $v_\mu$  (the corresponding constraints on the Wilson coefficients of  $\bar{c}\sigma_{\mu\nu}b$  are not sufficiently predictive to relate  $b'_i$  and  $c'_i$ ). However, the condition  $v^\mu J'_\mu = 0$  gives the following constraints for  $c'_i(\mu)$  and  $b'_i(\mu)$  separately

$$c'_0(\mu) = -c'_1(\mu), \quad b'_1(\mu) = b'_2(\mu), \quad b'_3(\mu) + b'_4(\mu) + b'_6(\mu) = 0. \quad (20)$$

The symmetry relation (9) follows from taking an appropriate linear combination of matrix elements of  $J_\mu$  and  $J'_\mu$  between hadronic states  $\langle \pi(p') | J_\mu + \kappa_1 J'_\mu | \bar{B}(p) \rangle$ , with  $\kappa_1$  to be chosen as explained below. In the full theory this matrix element can be expressed in terms of the form factors (1), (2)

$$\begin{aligned} \langle \pi(p') | J_\mu + \kappa_1 J'_\mu | \bar{B}(p) \rangle = \\ [f_+ - \kappa_1(m_B - v \cdot p')s](p + p')_\mu + [f_- + \kappa_1(m_B - v \cdot p')s](p - p')_\mu \end{aligned} \quad (21)$$

The same matrix element is given in the effective theory by

$$\begin{aligned} \langle \pi(p') | J_\mu + \kappa_1 J'_\mu | \bar{B}(p) \rangle = & (c_0 + \kappa_1 c'_0) \langle \bar{q} \gamma_\mu h_v \rangle + (c_1 + \kappa_1 c'_1) \langle \bar{q} v_\mu h_v \rangle \\ & + \frac{1}{2m_b} \left\{ \sum_{a=\text{kin,mag}} c_a(\mu) (c_0(\mu) + \kappa_1 c'_0(\mu)) \langle T \{ \bar{q} \gamma_\mu h_v, i\mathcal{L}_a \} \rangle \right. \\ & \left. + c_a(\mu) (c_1(\mu) + \kappa_1 c'_1(\mu)) \langle T \{ \bar{q} v_\mu h_v, i\mathcal{L}_a \} \rangle + \sum_{i=1}^6 (b_i + \kappa_1 b'_i) \langle \mathcal{J}_\mu^i \rangle \right\} \end{aligned} \quad (22)$$

At tree level, the combination of Wilson coefficients  $c_0 + \kappa_1 c'_0$  multiplying the matrix element of  $\bar{q}\gamma_\mu h_v$  vanishes (with  $\kappa_1 = 1$ ). Since the contribution of  $\bar{q}v_\mu h_v$  is proportional to  $p_\mu$ , it follows that the combination of form factors multiplying  $p'_\mu$  in (21) is suppressed by  $\Lambda/m_b$ . This was the original observation of [2] leading to the symmetry relation (P-1) (9). From (15) and (17) one can see that  $c_0 + c'_0 = 0$  is accidentally satisfied also at one-loop order, and a correction appears first at two-loop order. We will take in the following

$$\kappa_1 = -\frac{c_0(m_b)}{c'_0(m_b)} = 1 + (4.69 - 0.34n_\ell + 0.04) \left( \frac{\alpha_s(m_b)}{\pi} \right)^2 \quad (23)$$

such that this cancellation persists to any order in  $\alpha_s(m_Q)$ . Furthermore, this ensures also the cancellation of the nonlocal power-suppressed terms  $\langle \mathcal{L}_a, \bar{q}\gamma_\mu h_v \rangle$ . The remaining dimension-4 local terms can be easily expressed in terms of leading order form factors plus the additional form factors  $\delta_\pm(y)$  defined as

$$\langle \pi(p', \epsilon) | \bar{q} i D_\mu h_v | \bar{B}(v) \rangle = \delta_+(y)(p_\mu + p'_\mu) + \delta_-(y)(p_\mu - p'_\mu). \quad (24)$$

The equation of motion for the  $h_v$  field gives one constraint among these form factors ( $m_B + v \cdot p'$ ) $\delta_+(y) + (m_B - v \cdot p')$  $\delta_-(y) = 0$ , such that only one of them is independent. Inserting this into (22) and comparing the coefficient of  $p'_\mu$  on both sides, one finds the improved version of the symmetry relation (9)

$$(P-1') : \quad f_-(y) + \kappa_1 m_B s(y) = -\frac{1}{m_B}(\delta_+ - \delta_-) - (\bar{\Lambda} - \kappa_1 v \cdot p')s(y) + \mathcal{O}(m_b^{-3/2}) \quad (25)$$

We used on the right-hand side of (22) the tree level values of the Wilson coefficients  $b_i^{(0)}(m_b)$  (15), (17), which is sufficient for the accuracy intended. The corrections to (25) are suppressed by  $\frac{\Lambda^2}{m_Q^2}$  relative to the leading terms in (9), and come from matrix elements of dimension-5 operators which were neglected in (22).

Similar improved relations can be proved for the  $\bar{B} \rightarrow V$  form factors. The first symmetry relation (10) follows again from taking the matrix element

$$\langle \rho(p', \epsilon) | J_\mu + \kappa_1 J'_\mu | \bar{B}(v) \rangle = - \left( 2g + \kappa_1 \frac{1}{m_B} (g_+ - g_-) \right) i \varepsilon_{\mu\nu\lambda\sigma} \epsilon_\nu^* p_\lambda p'_\sigma. \quad (26)$$

The same quantity can be expressed in the effective theory with the help of (13) as

$$\begin{aligned} \langle \rho | J_\mu + \kappa_1 J'_\mu | \bar{B} \rangle &= (c_0(\mu) + \kappa_1 c'_0(\mu)) \langle \bar{q}\gamma_\mu h_v \rangle \\ &+ \frac{1}{2m_b} \left\{ \sum_{a=\text{kin, mag}} c_a(\mu) (c_0(\mu) + \kappa_1 c'_0(\mu)) \langle T \{ \bar{q}\gamma_\mu h_v, i \mathcal{L}_a \} \rangle + \sum_{i=1}^6 (b_i + \kappa_1 b'_i) \langle \mathcal{J}_\mu^i \rangle \right\} \end{aligned} \quad (27)$$

The matrix element of  $\bar{q}v_\mu h_v$  vanishes for this case by Lorentz invariance. The nonlocal  $1/m_b$  contributions cancel as before, leaving only local dimension-4 operators. They can be easily expressed in terms of leading order form factors plus one additional form factor  $\mathcal{D}(y)$  defined as

$$\langle V(p', \epsilon) | \bar{q} i D_\mu b | \bar{B}(v) \rangle = \mathcal{D}(y) i \varepsilon_{\mu\nu\lambda\sigma} \epsilon_\nu^* p_\lambda p'_\sigma. \quad (28)$$

From (27) one finds now the following result for the corrected symmetry relation (10)

$$\kappa_1(g_+ - g_-) + 2m_B g(y) = - \left( (g_+ + g_-) + \frac{\bar{\Lambda}}{m_B}(g_+ - g_-) + 2\mathcal{D}(y) \right) + \mathcal{O}(m_b^{-3/2}). \quad (29)$$

Using the leading order symmetry relations (11), (12) on the right-hand side, this can be put in a form similar to (10)

$$(V-1') : \quad \kappa_1(g_+ - g_-) + 2m_B g = -2(m_V y - \bar{\Lambda})g(y) - \frac{1}{m_B}f(y) - 2\mathcal{D}(y) + \mathcal{O}(m_b^{-3/2}). \quad (30)$$

The remaining two relations (11) and (12) can be derived in a similar way, starting with the two currents  $J_{5\mu} = \bar{q}i\sigma_{\mu\nu}v^\nu\gamma_5 b$  and  $J'_{5\mu} = (g_{\mu\nu} - v_\mu v_\nu)\bar{q}\gamma^\nu\gamma_5 b$ . Both these currents can be matched onto HQET operators as in (13)

$$J_{5\mu}^{(\eta)} = \tilde{c}_0^{(\eta)}(\mu)\bar{q}\gamma_\mu\gamma_5 h_v + \tilde{c}_1^{(\eta)}(\mu)\bar{q}v_\mu\gamma_5 h_v + \frac{1}{2m_b} \sum_{i=1}^6 \tilde{b}_i^{(\eta)}(\mu)\mathcal{J}_{5\mu}^i, \quad (31)$$

where the dimension-4 operators appearing on the right-hand side are defined analogously to (14) with the substitution  $h_v \rightarrow \gamma_5 h_v$ . The constraints  $v^\mu J_{5\mu} = v^\mu J'_{5\mu} = 0$  give relations among the Wilson coefficients analogous to (20)

$$\tilde{c}_0^{(\eta)}(\mu) = \tilde{c}_1^{(\eta)}(\mu), \quad \tilde{b}_1^{(\eta)}(\mu) + \tilde{b}_2^{(\eta)}(\mu) = 0, \quad \tilde{b}_3^{(\eta)}(\mu) + \tilde{b}_4^{(\eta)}(\mu) + \tilde{b}_6^{(\eta)}(\mu) = 0 \quad (32)$$

The matching conditions for the current  $J_{5\mu} = \bar{q}i\sigma_{\mu\nu}v^\nu\gamma_5 b$  are

$$\begin{aligned} \tilde{c}_0(m_b) = \tilde{c}_1(m_b) &= 1 - \frac{\alpha_s C_F}{\pi} - (16.09 - 1.13n_\ell + 0.13) \left( \frac{\alpha_s}{\pi} \right)^2 \\ \tilde{b}_1(m_b) &= 1, \tilde{b}_2(m_b) = -1, \quad \tilde{b}_{3-6}(m_b) = 0, \end{aligned} \quad (33)$$

and for  $J'_{5\mu} = (g_{\mu\nu} - v_\mu v_\nu)\bar{q}\gamma^\nu\gamma_5 b$

$$\begin{aligned} \tilde{c}'_0(m_b) = \tilde{c}'_1(m_b) &= 1 - (10.88 - 0.77n_\ell + 0.11) \left( \frac{\alpha_s}{\pi} \right)^2 \\ \tilde{b}'_1(m_b) = \tilde{b}'_2(m_b) &= -1, \quad \tilde{b}'_{3-6}(m_b) = 0. \end{aligned} \quad (34)$$

The values of  $\tilde{c}_{0,1}^{(\eta)}(m_b)$  quoted above correspond to the 't Hooft-Veltman scheme used in [13].

The two symmetry relations (11) and (12) are obtained by taking again a linear combination of the matrix elements of  $J_{5\mu}$  and  $J'_{5\mu}$ , with  $\kappa_5$  to be determined below

$$\begin{aligned} \langle \rho(p', \varepsilon) | J_{5\mu} - \kappa_5 J'_{5\mu} | \bar{B}(p) \rangle &= \varepsilon_\mu^* (g_+(m_B + v \cdot p') + g_-(m_B - v \cdot p') - \kappa_5 f) \\ &+ (v \cdot \varepsilon^*)(p_\mu + p'_\mu) \left( -g_+ - m_B h(m_B - v \cdot p') + \frac{1}{2m_B} \kappa_5 [f - m_B(m_B - v \cdot p')(a_+ - a_-)] \right) \\ &+ (v \cdot \varepsilon^*)(p_\mu - p'_\mu) \left( -g_- + m_B h(m_B + v \cdot p') + \frac{1}{2m_B} \kappa_5 [f + m_B(m_B + v \cdot p')(a_+ - a_-)] \right). \end{aligned} \quad (35)$$

This relation involves a subtle point which deserves special attention. Usually, the matrix element  $\langle \rho | J_{5\mu} | \bar{B} \rangle$  is computed in terms of the form factors of the current  $\bar{q}\sigma_{\mu\nu}b$  with the help of the relation  $\sigma_{\mu\nu}\gamma_5 = \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$  (corresponding to the convention  $\varepsilon^{0123} = 1$ ). Beyond

tree-level, this relation is only true in the 't Hooft-Veltman scheme for  $\gamma_5$ . In any other scheme, there are calculable  $\mathcal{O}(\alpha_s)$  corrections to the relation between the matrix elements  $\langle \bar{q} \sigma_{\mu\nu} b \rangle$  and  $\langle \bar{q} \sigma_{\mu\nu} \gamma_5 b \rangle$ , which cancel the scheme dependence in the individual Wilson coefficients  $\tilde{c}_{0,1}^{(i)}(m_b)$ . For this reason, the use of the 't Hooft-Veltman scheme will be understood throughout in the following.

The combination of matrix elements (35) is given in the effective theory by

$$\begin{aligned} \langle \rho(p', \varepsilon) | J_{5\mu} - \kappa_5 J'_{5\mu} | \bar{B} \rangle &= (\tilde{c}_0 - \kappa_5 \tilde{c}'_0) (\langle \bar{q} \gamma_\mu \gamma_5 h_v \rangle + \langle \bar{q} v_\mu \gamma_5 h_v \rangle) \\ &+ \frac{1}{2m_b} \left\{ \sum_{a=\text{kin, mag}} c_a (\tilde{c}_0 - \kappa_5 \tilde{c}'_0) \langle T \{ \bar{q} (\gamma_\mu + v_\mu) \gamma_5 h_v, i \mathcal{L}_a \} \rangle + \sum_{i=1}^6 (\tilde{b}_i - \kappa_5 \tilde{b}'_i) \langle J_{5\mu}^i \rangle \right\} \end{aligned} \quad (36)$$

Using the tree level values of the Wilson coefficients, the contribution of the dimension-3 operators vanishes with  $\kappa_5 = 1$ , which gives the leading order symmetry relations (11) and (12). The same cancellation will hold true to all orders in  $\alpha_s(m_Q)$  provided that  $\kappa_5$  is chosen as  $\kappa_5 = \tilde{c}_0(m_b)/\tilde{c}'_0(m_b) = 1 - \alpha_s(m_b)C_F/\pi + \mathcal{O}(\alpha_s^2)$ .

The remaining matrix elements of dimension-4 operators can be expressed in terms of leading order form factors (3), (5) and 3 additional form factors  $\mathcal{D}_1, \mathcal{D}_\pm$  defined as

$$\langle V(p', \varepsilon) | \bar{q} i D_\mu \gamma_5 h_v | \bar{B}(v) \rangle = \mathcal{D}_1 \varepsilon_\mu^* + \mathcal{D}_+(\varepsilon^* \cdot p)(p_\mu + p'_\mu) + \mathcal{D}_-(\varepsilon^* \cdot p)(p_\mu - p'_\mu). \quad (37)$$

The equation of motion for the heavy quark field  $iv \cdot D h_v = 0$  implies a relation among these form factors, such that only two of them are independent.

Using standard HQET methods, one finds the following two generalized symmetry relations, including subleading corrections

$$\begin{aligned} (g_+ + g_-)m_B + v \cdot p'(g_+ - g_-) - \kappa_5 f &= \\ (g_+ + g_-)(\bar{\Lambda} - v \cdot p') + \frac{1}{m_B}(g_+ - g_-)(\bar{\Lambda} v \cdot p' - m_V^2) &+ \frac{2}{m_B} \mathcal{D}_1 + \mathcal{O}(m_b^{-3/2}) \end{aligned} \quad (38)$$

and

$$\begin{aligned} g_+ - g_- + 2m_B^2 h + \kappa_5 m_B(a_+ - a_-) &= \\ \frac{\bar{\Lambda}}{m_B}(g_+ - g_-) - 2m_B h(\bar{\Lambda} - v \cdot p') - 2(\mathcal{D}_+ - \mathcal{D}_-) &+ \mathcal{O}(m_b^{-3/2}). \end{aligned} \quad (39)$$

The first relation can be put in a form similar to (11) by inserting the lowest order result (30) for  $g_+ - g_-$ . One obtains in this way the improved version of the symmetry relation (11), including subleading  $\Lambda/m_b$  and hard gluon corrections

$$\begin{aligned} (\text{V-2}') : \quad g_+ + g_- - 2m_V y g - \kappa_5 \frac{1}{m_B} f &= \\ -2 \frac{m_V}{m_B} (\bar{\Lambda} y - m_V) g(y) + \frac{\bar{\Lambda}}{m_B^2} f + \frac{2}{m_B^2} (m_V m_B y \mathcal{D}(y) &+ \mathcal{D}_1(y)) + \mathcal{O}(m_b^{-5/2}) \end{aligned} \quad (40)$$

The analog of the symmetry relation (12) is obtained in a similar fashion from (39) and reads

$$\begin{aligned} (\text{V-3}') : \quad \kappa_5(a_+ - a_-) + 2m_B h - 2g &= 2 \left( \frac{m_V}{m_B} y - 2 \frac{\bar{\Lambda}}{m_B} \right) g(y) + \frac{1}{m_B^2} f(y) \\ -2h(\bar{\Lambda} - m_V y) + \frac{2}{m_B} (\mathcal{D}(y) - \mathcal{D}_+(y) &+ \mathcal{D}_-(y)) + \mathcal{O}(m_b^{-5/2}) \end{aligned} \quad (41)$$

In conclusion, the leading  $\Lambda/m_b$  corrections to the symmetry relation (9) for  $\bar{B} \rightarrow P$  decays are expressed in terms of one additional form factor  $\delta_+ - \delta_-$ , while the symmetry relations for  $\bar{B} \rightarrow V$  decays (10)-(12) require three additional form factors  $\mathcal{D}(y)$ ,  $\mathcal{D}_1(y)$  and  $\mathcal{D}_+(y) - \mathcal{D}_-(y)$ . All these four quantities are matrix elements of the two local operators in the effective theory  $\bar{q}iD_\mu(\gamma_5)h_v$ . The relative simplicity of this result can be appreciated by noting that 22 new matrix elements, both local and nonlocal, are required for a complete  $1/m_Q$  expansion of the individual form factors in  $\bar{B} \rightarrow V$  decays [8]. We note also that the local nature of these corrections should make their computation on the lattice feasible. In the following, we will estimate them using various approximation schemes and models.

The form factors  $\delta_\pm(y)$  (24) relevant for the  $\bar{B} \rightarrow P$  decays can be computed in the soft pion limit if  $P$  is one of the members of the Goldstone bosons octet. We use for this purpose the chiral perturbation theory for heavy hadrons developed in [14]. To lowest order in the chiral expansion, there is a unique operator which is a realization of  $\bar{q}\Gamma iD_\mu h_v$ . Such an operator must satisfy the following conditions: a) transforms as  $(\bar{3}_L, 1_R) + (1_L, \bar{3}_R)$  under  $SU_L(3) \times SU_R(3)$ , b) transforms in the same way under heavy quark spin rotations as  $\bar{q}\Gamma h_v$  and c) vanishes upon contraction with  $v^\mu$ . The only operator with these properties is

$$\bar{q}\Gamma iD_\mu h_v \rightarrow \frac{i}{2}\alpha \text{Tr} [(\gamma_\mu + v_\mu)\Gamma H_b(\xi_{ba}^\dagger + \xi_{ba})] + \frac{i}{2}\alpha \text{Tr} [(\gamma_\mu + v_\mu)\gamma_5\Gamma H_b(\xi_{ba}^\dagger - \xi_{ba})], \quad (42)$$

where the Goldstone bosons are described by the field  $\xi = \exp(\frac{i}{f_\pi}M)$ , and the heavy mesons  $\bar{B}_a^{(*)}$  are contained in the superfield  $H_a$  defined as in [1]. The low-energy constant  $\alpha$  can be determined by taking the  $\bar{B}$ -to-vacuum matrix element of this operator with  $\Gamma = \gamma^\mu \gamma_5$ , which gives  $\alpha = -\frac{1}{6}f_B m_B \bar{\Lambda}$ .

The form factors  $\delta_\pm(y)$  introduced in (24) can be computed now to the leading order in the chiral expansion. There are two contributions involving the operator (42), the direct and the pole graphs. Their computation gives for the  $\bar{B}_d \rightarrow \pi^+$  form factors (this is multiplied with  $1/\sqrt{2}$  for the  $B^- \rightarrow \pi^0$  case)

$$\delta_+ - \delta_- = \frac{g\bar{\Lambda}f_B}{3f_\pi} \cdot \frac{m_B}{v \cdot p' + \Delta} = \frac{(0.5 \pm 0.1) \text{ GeV}^2}{v \cdot p' + \Delta} \quad (43)$$

with  $g$  the  $BB^*\pi$  coupling and  $\Delta = m_{B^*} - m_B$  the hyperfine splitting in the  $B$  meson system. This matrix element can be also extracted from the current algebra calculation of [7]. We used in the numerical estimate the recent CLEO measurement [15]  $g = 0.59 \pm 0.12$ ,  $\bar{\Lambda} = 350$  MeV and  $f_B = 180$  MeV.

For the  $\bar{B} \rightarrow V$  case, no predictions can be obtained from chiral symmetry. Some information can be obtained however from the constituent quark model. We will show in the following that the form factor  $\mathcal{D}(y)$  defined in (28) vanishes exactly in the quark model, which suggests that its real value might be very small. To see this, note that in the quark model the vector meson state is in general a mixture of  $S$ -wave and a  $D$ -wave states ( $^3S_1$  and  $^3D_1$  in spectroscopic notation  $^{2S+1}L_J$ ), whereas the  $B$  meson state contains only a  $S$ -wave component  $^1S_0$

$$|V(\vec{0}, m_z = +1)\rangle = c_S \phi_S(|\vec{k}|) |\uparrow\uparrow\rangle \quad (44)$$

$$+ c_D \phi_D(|\vec{k}|) \left( \frac{1}{\sqrt{3}} Y_{2,+2}(\hat{k}) |\downarrow\downarrow\rangle + \frac{1}{\sqrt{6}} Y_{2,+1}(\hat{k}) \cdot \frac{1}{\sqrt{2}} |\uparrow\downarrow + \downarrow\uparrow\rangle - \frac{1}{\sqrt{2}} Y_{2,0}(\hat{k}) |\uparrow\uparrow\rangle \right)$$

$$|\bar{B}(v)\rangle = \phi_B(|\vec{k}|) \frac{1}{\sqrt{2}} |\uparrow\downarrow - \downarrow\uparrow\rangle \quad (45)$$



$\mathcal{D}(y)$  is given by the product of an orbital overlap integral and a spin matrix element. The latter is only nonvanishing for the  $D$ -wave component of the vector meson. However the total result vanishes because of the orthogonality of the spin wavefunctions

$$\begin{aligned} \mathcal{D}(y) &\propto \langle V(\vec{q}, m_z = +1) | (\vec{e}_{+1} \times \vec{q}) \cdot \vec{k} | \bar{B}(v) \rangle \\ &\propto \int d^3\vec{k} \phi_D(|\vec{k} + \vec{q}|) Y_{2,+1}^*(\hat{k}) (\vec{e}_{+1} \cdot \vec{k}) \phi_B(|\vec{k}|) \times \langle \uparrow\downarrow + \downarrow\uparrow | \uparrow\downarrow - \downarrow\uparrow \rangle = 0. \end{aligned} \quad (46)$$

At large recoil  $E_V \gg \Lambda_{QCD}$ , the form factor  $\mathcal{D}(y)$  can be also computed in a light-cone expansion in  $1/E_V$ . This kinematical region is at best appropriate to  $\bar{B} \rightarrow V$  decays at low values of  $q^2$ , which is strictly speaking outside the validity domain of (30). Still, such a calculation is instructive as an order of magnitude estimate. In this approach, the matrix element (28) is given by a diagram with a hard gluon exchanged between the current and the spectator quark. We will consider in the following the  $\bar{B} \rightarrow \rho$  case, for which one obtains at lowest order in  $1/E_\rho$

$$\mathcal{D}(E_\rho) = -\frac{\pi\alpha_s N_c C_F}{E_\rho} f_B f_\rho^\perp \int_0^\infty dk_+ \frac{\phi_B(k_+)}{k_+} \int_0^1 du \frac{\phi_\perp(u)}{1-u}, \quad (47)$$

where  $\phi_B(k_+)$  is the leading twist  $B$  meson light-cone wave function, normalized according to  $\int_0^\infty dk_+ \phi_B(k_+) = 1$ . For the  $\rho$  meson we kept only the twist-2 chiral-odd wavefunction  $\phi_\perp(u)$  [16] appropriate for a transversely polarized meson. Since we consider all form factors to be renormalized at  $\mu = m_b$ , we will be using this scale [23] in the numerical evaluation of (47). Using  $f_B = 180$  MeV,  $f_\rho^\perp(m_b) = 144$  MeV [16], together with the asymptotic expression  $\phi_\perp(u) = 6u(1-u)$  and the inverse moment of the  $B$  wave function  $\langle (k_+)^{-1} \rangle = 3$  GeV $^{-1}$ , we find  $\mathcal{D}(E_\rho) \simeq -(0.9 \text{ GeV})/E_\rho$ .

We turn now to a brief discussion of the implications of these relations. The most important result following from (10), (11) was the possibility of extracting the radiative form factor  $g_+(y)$  from semileptonic decays data. Such analyses have been presented in [10, 11, 12]. This relation can be now extended to next-to-leading order in  $\Lambda/m_b$  by combining (30) with (11)

$$g_+(y) = -(m_B - \bar{\Lambda})g(y) - \mathcal{D}(y) + \mathcal{O}(m_b^{-3/2}) \quad (48)$$

Note that including the subleading term in (30) is essential, since it is formally of the same order in  $\Lambda/m_b$  as the other terms kept.

Numerically, the subleading terms in (30) can be significant. For illustration, consider the  $D \rightarrow K^*$  form factors at  $y = 1$ , for which numerical results are available from the E791 Collaboration  $g(1) = -(0.49 \pm 0.04) \text{ GeV}^{-1}$ ,  $f(1) = (1.9 \pm 0.1) \text{ GeV}$  [17]. From (48) one finds  $g_+^{D \rightarrow K^*}(1) = (0.74 \pm 0.06)$ , where we used  $\bar{\Lambda} = 350$  MeV and  $\mathcal{D}(1) = 0$ . The subleading correction in (30) contributed about 25% to this result. On the other hand, the perturbative effect of hard gluons enters only at two-loop order (through  $\kappa_1$ , given in Eq. (23)), and therefore can be safely neglected.

Assuming that the vector form factor  $g^{D \rightarrow V}(y)$  has been measured, the relation (48) allows the extraction of  $g_+^{D \rightarrow V}(y)$  with accuracy of order  $\Lambda^2/m_c^2$ . Neglecting experimental errors, the theoretical uncertainty in such a determination of  $g_+^{D \rightarrow V}(y)$  could be made as small as 10%. A precise knowledge of this form factor is relevant for a good control of the short-distance matrix element in rare  $D$  decays [19].

The corrected symmetry relations we derived in this Letter can be expected to help reduce the theoretical uncertainties in constrained extractions of form factors from experimental data, such as those presented in [10, 11, 12, 18], and in determinations of CKM parameters with the help of  $SU(3)$  and heavy quark symmetry [20, 21, 22]. We will present details of such an investigation in a separate publication.

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